It should be noted that it is particularly important to satisfy inequality (5) in solving nonlinear problems. Thus, the solution of a nonlinear heat-conduction problem showed that the iterations do not converge if inequality (5) is not satisfied, where a, b, and c are taken as the extreme values of the corresponding coefficients of the equation.

LITERATURE CITED

1. S. K. Godunov and V. S. Ryaben'kii, Difference Schemes [in Russian], Nauka (1973).
2. R. D. Richtmyer and K. W. Morton, Difference Methods for Initial Value Problems, 2nd. ed., Wiley-Interscience, New York (1967).
3. M. E. Weber, "Improving the accuracy of Crank—Nicolson numerical solutions of the heat-conduction equation," Trans. ASME, J. Heat Transfer, 91, 189 (1969).
4. V. I. Barsukov, V. D. Vilenskii, V. M. Paskonov, and V. I. Taratorin, High-Temperature Boundary Layer in Air [in Russian], MGU (1971).

THE STRESS-DIFFUSION EFFECT IN HETEROGENEOUS LIQUID STREAMS

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An analysis is made of the effect of the shear stresses in the average stream on the coefficient of molar diffusion of a passive impurity in a turbulent liquid stream.

In a study of the slow motion of a specularly reflecting sphere in a heterogeneous stream of rarefied gas a dependence of the coefficient of dynamic friction on the tensor of shear stress in the stream (the stress-phoretic effect) was obtained in [1]. In this case the transfer processes are determined by the following tensor expression for the coefficient of Brownian diffusion:

$$D_{ij} = D^0(\delta_{ij} - \alpha\sigma_{ij}/p), \tag{1}$$

where D^0 is the Einstein coefficient of Brownian diffusion; σ_{ij} are the coefficients of the reduced stress tensor; p is the hydrostatic pressure; δ_{ij} is the Kronecker symbol; α is a numerical coefficient. Analogous results follow within the framework of the kinetic theory in an analysis of the motion of heavy impurity particles in a nonequilibrium gas [2].

In the present report it is shown that a similar dependence of the diffusion coefficient on the stress tensor in a stream also occurs in the case of the turbulent diffusion of a passive impurity in a shear stream (the stress-diffusion effect).

Let $v(t, x; \omega)$ be the random vector field of the velocity of a turbulent stream of incompressible liquid; $\vartheta(t, x; \omega)$ be the scalar field of the concentration of a passive impurity in it, which satisfies the transfer equation [3]

$$\frac{\partial}{\partial t}\vartheta(t, x; \omega) = -\frac{\partial}{\partial x_i} v_i(t, x; \omega)\vartheta(t, x; \omega). \tag{2}$$

Let us examine the derivation of the equation of convective diffusion for $\bar{\vartheta}$ on the assumption of the statistical independence of the initial distribution $\vartheta(0, x; \omega) = \vartheta_0(x; \omega)$ from the velocity fluctuations in the in the turbulent stream and in the approximation of a Gaussian velocity field. We take $v_i = \bar{v}_i + v'$ and $\bar{v}_i' = 0$ and introduce the designation

$$\Omega(t; \omega) = -U^{-1}(t)\frac{\partial}{\partial x_i} v_i'(t, x; \omega)\, U(t),\ U(t) = \exp\left[-t\frac{\partial}{\partial x_i}\bar{v}_i(x)\right], \tag{3}$$

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with allowance for which we write the formal solution of (2) in the form

$$\vartheta(t, x; \omega) = U(t) \exp\left[\int_0^t dt_1 \Omega(t_1; \omega)\right] \vartheta_0(x; \omega) . \tag{4}$$

We average this expression over the ensemble of fluctuations v_i' of the velocity field and the initial distribution ϑ_0 of the impurity, allowing for their statistical independence, and using the definition of the semiinvariants of the correlation functions of the random operator $\Omega(t; \omega)$:

$$\bar{\vartheta}(t, x) = U(t) \left\langle \exp\left[\int_0^t dt_1 \Omega(t_1; \omega)\right] \right\rangle \bar{\vartheta}_0(x) = U(t) \exp\left[\int_0^t dt_1 \int_0^{t_1} dt_2 \left\langle \Omega(t_1; \omega) \Omega(t_2; \omega) \right\rangle \right] \bar{\vartheta}_0(x) . \tag{5}$$

The closed equation of turbulent diffusion follows from (5) when the time derivative is calculated with allowance for Eqs. (3) and the condition of incompressibility of the stream

$$\frac{\partial \bar{\vartheta}}{\partial t} + \bar{\vartheta}_i \frac{\partial \bar{\vartheta}}{\partial x_i} = - \frac{\partial}{\partial x_i} \bar{j}_i , \tag{6}$$

where the diffusional flux $\bar{j}_i$ has the form

$$\bar{j}_i(t, x) = - \int_0^t dt_1 \left\langle v_i'(t, x) U(t - t_1) v_i'(t_1, x) \right\rangle \frac{\partial}{\partial x_j} U^{-1}(t - t_1) \bar{\vartheta}(t, x) .$$

We transform the right side of this expression as follows:

$$\bar{j}_i(t, x) = - \int_0^t dt_1 \int dy^3 \left\langle v_i'(t_1, y) v_j'(t, x) \right\rangle U(t - t_1) \delta(x - y) \frac{\partial}{\partial x_j} U^{-1}(t - t_1) \bar{\vartheta}(t, x) \tag{7}$$

and allow for the relation

$$\frac{\partial}{\partial x_j} U^{-1}(t - t_1) \bar{\vartheta} = U^{-1}(t - t_1) \left[\frac{\partial}{\partial x_j} + (t - t_1) \frac{\partial \bar{v}_k}{\partial x_j} \frac{\partial}{\partial x_k}\right] \bar{\vartheta} . \tag{8}$$

We note that

$$U(t - t_1) \delta(y - x) U^{-1}(t - t_1) = \delta[y - x^*(t, t_1)], \tag{9}$$

where $x_i^*(t, t_1) = U(t - t_1) x_i U^{-1}(t - t_1)$. The function $x_i^*(t, t_1)$ satisfies the condition $x_i^*(t_1, t_1) = x_i$ and the equation

$$\frac{dx_i^*(t, t_1)}{dt} = U(t - t_1) \bar{v}_i(x) U^{-1}(t - t_1) = \bar{v}_i[x^*(t, t_1)] ,$$

which follows directly from the definition of $x_i^*(t, t_1)$ and Eq. (3). Consequently, $x^*(t, t_1)$ represents the trajectory of the motion of a passive particle with the averaged stream velocity $\bar{v}(x)$ from the point x at the time t_1. Substituting (8) and (9) into (7), after integrating over y we obtain

$$\bar{j}_i(t, x) = - \int_0^t dt_1 \left\langle v_i'(t, x) v_j'[t_1, x^*(t, t)] \right\rangle \times \left[\frac{\partial \bar{\vartheta}(t, x)}{\partial x_j} + (t - t_1) \frac{\partial \bar{v}_k(x)}{\partial x_j} \frac{\partial \bar{\vartheta}(t, x)}{\partial x_k}\right] \tag{10}$$

or, designating

$$\beta_{ij}^*(t, x) = \int_0^t dt_1 \left\langle v_i'(t, x) v_j'[t_1, x^*(t, t_1)] \right\rangle , \tag{11}$$

$$\gamma_{ij}^{*}(t,x) = \int_0^t dt_1 (t - t_1) \langle v_i'(t,x) v_j'[t_1, x^*(t,t_1)] \rangle , \tag{12}$$

allowing only for the symmetrical part of the tensor $\partial \bar{v}_k/\partial x_j$ in (10), and introducing the shear-stress tensor $\sigma_{kj} = -2\mu_t\{\partial \bar{v}_k/\partial x_j\}_s$, we obtain

$$\bar{j}_i(t,x) = -\beta_{ij}^{*}(t,x)\frac{d\bar{\vartheta}}{\partial x_j} + \frac{1}{\mu_t}\gamma_{ik}^{*}(t,x)\sigma_{kj}\frac{\partial\bar{\vartheta}}{\partial x_j} . \tag{13}$$

The second term in (13) explicitly allows for the effect of the influence of the shear stresses in the averaged stream on the diffusion of the passive impurity. Since the correlations of the field $v'(t, x; \omega)$ are important over intervals $t - t_1$ not exceeding the time τ_v of correlation of pulsations in the turbulent stream and over distances $x - x^*(t, t_1)$ not exceeding the radius R of the spatial correlations, in the case of $\bar{u}\tau_v \ll R$ ($\bar{u}$ is the characteristic velocity of the averaged stream) one can replace $x^*(t, t_1)$ by $x^*(t, t) = x$ in Eqs. (11) and (12). Equation (13) can be considered as the basis for the construction of semiphenomenological theories of turbulent diffusion.

Let us further consider the process of diffusion in time scales larger than τ_v, neglecting the time dependence of β_{ij} and γ_{ij} (using their values at $t = \infty$). Since the time correlations of the velocity pulsations die out faster than $1/(t - t_1)$, the assumption that $\gamma_{ij}(t, x)$ is finite as $t \to \infty$ is obvious. Such asymptotic behavior corresponds to a Markov approximation of particle motion in coordinate space but differs from the usual approximation [3] corresponding to the approximation of a field $v'(t, x; \omega)$ which is δ-correlated in time. We assume that $\beta_{ij}(\infty, x) = T_{ij}\overline{v'^2} = D^t_{ij}$ is the coefficient of molar diffusion, T_{ij} is the correlation time of the velocity pulsations, and $\tau_v = \max T_{ij}$) and in the spirit of the phenomenological theory we represent $\gamma_{ji}(\infty, x)$ in the form

$$\gamma_{ij}(\infty, x) = \alpha T_{ii} D^t_{ij}, \tag{14}$$

where α is a numerical coefficient. With allowance for (14) we represent (13) in the form

$$\bar{j}_i(t,x) = -D_{ij}\frac{\partial\bar{\vartheta}}{\partial x_j} = -D^t_{ij}\left[\delta_{kj} - \frac{\alpha}{\mu_t}T_{ik}\delta_{kj}\right]\frac{\partial\bar{\vartheta}}{\partial x_j} . \tag{15}$$

Thus, the expression (15) for the coefficient D_{ij} in the equation (6) of convective diffusion in a heterogeneous turbulent stream is formally analogous to the coefficient of Brownian diffusion (1). However, if we extend the analogy between the physical situation being considered here and Brownian diffusion on the basis of Eq. (2), where $v(t, x; \omega)$ is the velocity of a Brownian particle in a heterogeneous medium, then in the case of a dependence of the coefficient of dynamic friction of a Brownian particle on the stress tensor [2] one should use Eq. (1) in calculating the coefficient $\beta_{ij}(t, x)$. It therefore reflects the dependence of the correlation function of the velocity of a Brownian particle on the stress tensor in the stream. The dependence of the coefficient of molar diffusion D^t_{ij} on the stress tensor of an averaged stream is an analog of the stress-phoretic effect in turbulent diffusion. In the case of Brownian motion (the velocity fluctuations are the result of thermal fluctuations of the medium), the second term in (13) is negligibly small in comparison with the first. It can prove to be important, however, in the case of macroscopic fluctuations in a turbulent stream. Here Eq. (15) reflects the effect of the interaction between the molar "mixing" of the passive impurity and its motion in a heterogeneous averaged stream.

LITERATURE CITED

1. W. G. H. Slinn and S. F. Shen, J. Stat. Phys., 3, No. 3 (1971).
2. O. A. Grechannyi, Inzh.-Fiz. Zh., 26, No. 6 (1974).
3. A. S. Monin and A. M. Yaglom, Statistical Fluid Mechanics [in Russian], Part 1, Nauka, Moscow (1965).